Problem Set 4 - Joint, Marginal and Conditional Distributions

ECO 104 (Section 7 and 9)
Instructor: Shaikh Tanvir Hossain
TA: Habiba Afroz

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## Joint Distributions for Discrete Random Variables

1. Suppose we have two random variables $X$ and $Y$ where $X$ represents gender and $Y$ represents average income of an EWU student. Usually we assume income is a continuous random variable, but suppose now income is discrete in three categories, so $Y$ can take values $10,000$ BDT, $20,000$ BDT, and $30,000$ BDT. Also assume $X = 1$ means female and $X = 0$ means male. Assume we have following joint distribution:

| | $Y = 10,000$ | $Y = 20,000$ | $Y = 30,000$ |
|---|---|---|---|
| $X = 1$ | 0.1 | 0.2 | 0.3 |
| $X = 0$ | 0.2 | 0.1 | 0.1 |

Answer following questions based on the above joint distribution.

(a) Find Marginal PMF of $X$, which we write with $f(x)$ and $Y$, which we write with $f(y)$.

(b) Find Marginal Expected Values of $X$, which we write with $\mathbb{E}(X)$ and $Y$, which we write with $\mathbb{E}(Y)$.

(c) Find Marginal Variances of $X$, which we write with $\mathbb{V}(X)$ and $Y$, which we write with $\mathbb{V}(Y)$.

(d) If we write conditional PMF of $Y$ given $X = 1$, which we write with $f(y|x = 1)$, write down the conditional PMF.

(e) How many conditional PMF can we write down in total?

(f) Find conditional expectation or conditional mean $\mathbb{E}(Y|X = 1)$ and $\mathbb{E}(Y|X = 0)$. What is the interpretation of these two quantities?

(g) Show Law of Total Expectation or Law of Iterated Expectations which says

$$\mathbb{E}(Y) = \mathbb{E}(\mathbb{E}(Y|X))$$

(h) Calculate the covariance and correlation from the joint table $\mathbb{C}\text{ov}(X, Y)$. What can you conclude?

(i) Are the two variables $X$ and $Y$ independent? Justify your answer.

(j) If you start from the Marginal PMF of $X$ and $Y$ can you now construct and new joint PMF of $X$ and $Y$ such that they are independent. Calculate also the Covariance and Correlation for this new joint PMF (you should get 0)

**Remarks:** You should look at Newbold et al. (2020) for more problems and if possible do more problems from there.

## References

Newbold, P., Carlson, W. L. & Thorne, B. M. (2020), *Statistics for Business and Economics*, 9th, global edn, Pearson, Harlow, England.